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Theoretical analysis of damping effects of SAW at solid/fluid interfaces

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Abstract : A theoretical of ideal and viscous fluid media is proposed to address the problem of modelling damping effects of Surface Acoustic Waves (SAW) at the interface between fluids and solids. It is based on the Fahmy-Adler eigenvalue representation of the elastic propagation problem, extended to provide the Green's function of the considered media. It takes advantage of previous efforts developed to numerically stabilize the Green's function computation process. The influence of acoustic radiation and viscosity on different kind of surface waves on various substrate is reported and discussed.

I. Introduction

Surface Acoustic Waves (SAW) can be excited at the surface of any solid material. These waves may exhibit elliptic as well as pure shear polarisation (case of isotropic media), but the practical case of wave propagation at the surface of anisotropic material generally yields any combination of wave polarisation, except along given crystal or symmetry axes [1]. True SAW are assumed to propagate without any losses along the guiding surface, providing a nice opportunity to manufacture low loss devices such as filters and resonators. On the other hand, it is also known that best quality factors and/or smallest insertion losses of SAW devices are obtained using package closed under vacuum to avoid leakage due to acoustic radiation in air. Also, the use of SAW devices for the development of sensors immersed in fluid media has been widely investigated. Theoretical analysis of SAW excitation and propagation under such working conditions requires the adaptation of existing simulation tools to provide a reliable description of the induced effects by the nature of the surrounding medium on the SAW device response.

In the proposed paper, the way the Green's function analysis and the harmonic admittance can be used in that matter is described. The mathematical developments required in that matter are exposed. Ideal fluids are first considered, but also viscous fluids (in the limit of Newtonian fluid assumption [2]) are simulated. Many cases are considered, for instance attenuation of Rayleigh waves due to radiation in air but also water damping of Lamb waves. The theoretical results are then discussed. The specific situation of pure shear waves as used in surface transverse waves (STW) and in acoustic plate mode (APM) devices is also regarded. These waves are generally assumed poorly affected by water and other weakly viscous fluids. The limit of this hypothesis is examined theoretically.

II. Theoretical fundaments

<u>II.1 Modelling the acoustic behaviour of ideal and viscous fluids</u>

The theoretical representation of acoustic waves in fluids is usually performed using a pressure formulation. Nevertheless, in order to easily derive the corresponding Green's function, a displacement formulation can be constructed as well. For any fluid, the independent elastic constants required for such a formulation reduce to one, i.e. C_{11} which is also equal to C_{12}, yielding C_{66}=0 consequently. According to [2], a shear effect in a fluid between a moving solid and a reference solid results in a linear stress proportional to the velocity gradient via a coefficient written η called shear viscosity or absolute viscosity of the fluid. In an isotropic homogeneous uncompressible Newtonian fluid, the stress is proportional to the linear strain ; this is called the Stokes low. In a very general approach [3], one should also consider a compressive viscosity factor written ζ but no relaxation phenomena (no specific time dependence), yielding the following formula :

$$T_{ij}=2j\omega(S_{ij}-\frac{1}{3}S_{kk}\delta_{ij})+\delta_{ij}(-P+j\omega\zeta S_{kk}) \quad (1)$$

where T_{ij} and S_{ij} respectively correspond to the stress and strain tensors, and ω is the angular frequency. The pressure is proportional to the displacement gradient via the compressibility factor. Practically, one can neglect the compressive viscosity contribution, resulting in the following expression of the stress :

$$T_{ij}=-(P+j\omega\frac{2}{3}\eta S_{kk})\delta_{ij}+2j\omega\eta S_{ij} \qquad (2)$$

One can remark that for η=0 (no viscosity effects in the fluid), eq.(2) reduces to the classical pressure equilibrium with no shear effects (T_{ij}=0 for $i\neq j$).

II.2 Fahmy-Adler formulation for viscous fluids

It is now explain how one can represent the propagation of acoustic waves in viscous fluids as an eigenvector problem conformably to the general description of Fahmy-Adler for solids. Without any loss of generality, one considers the propagation in the plane (x_1,x_3) and the dependence along x_2 is given by the system to solve, as shown further. We assume an harmonic dependence along time, omitting to report the implicit term $e^{j\omega t}$. We define the following state vector, mixing displacement and stress components in the propagation plane :

$$h=\left\langle \frac{T_{21}}{j\omega}\ \frac{T_{22}}{j\omega}\ \frac{T_{23}}{j\omega}\ u_1\ u_2\ u_3 \right\rangle \qquad (3)$$

In a very general approach, one should add the electrical potential and the electrical displacement vector component along x2, enabling one to represent dielectric viscous fluids. For the sake of simplicity, we only focus on the acoustic contribution. The propagation equations provide the first derivatives of the in-plane stresses versus x_2 as follows :

$$\frac{\partial}{\partial x_2}\begin{Bmatrix}T_{21}\\T_{22}\\T_{23}\end{Bmatrix}=-\rho\omega^2[I]\begin{Bmatrix}u_1\\u_2\\u_3\end{Bmatrix}-\frac{\partial}{\partial x_1}\begin{Bmatrix}T_{11}\\T_{12}\\T_{13}\end{Bmatrix}-\frac{\partial}{\partial x_3}\begin{Bmatrix}T_{31}\\T_{32}\\T_{33}\end{Bmatrix} \qquad (4)$$

We now need three more equation to establish the first derivatives of the displacement vs x_2. This is performed by developing eq.(2) as follows, using the notation $\sigma_{ij}=T_{ij}/j\omega$:

$$\begin{aligned}
\begin{Bmatrix}\sigma_{11}\\\sigma_{12}\\\sigma_{13}\end{Bmatrix}&=\frac{\partial}{\partial x_1}[A_{11}]\begin{Bmatrix}u_1\\u_2\\u_3\end{Bmatrix}+\frac{\partial}{\partial x_2}[A_{12}]\begin{Bmatrix}u_1\\u_2\\u_3\end{Bmatrix}+\frac{\partial}{\partial x_3}[A_{13}]\begin{Bmatrix}u_1\\u_2\\u_3\end{Bmatrix}\\
\begin{Bmatrix}\sigma_{21}\\\sigma_{22}\\\sigma_{23}\end{Bmatrix}&=\frac{\partial}{\partial x_1}{}^t[A_{12}]\begin{Bmatrix}u_1\\u_2\\u_3\end{Bmatrix}+\frac{\partial}{\partial x_2}[A_{22}]\begin{Bmatrix}u_1\\u_2\\u_3\end{Bmatrix}+\frac{\partial}{\partial x_3}[A_{23}]\begin{Bmatrix}u_1\\u_2\\u_3\end{Bmatrix} \qquad (5)\\
\begin{Bmatrix}\sigma_{31}\\\sigma_{32}\\\sigma_{33}\end{Bmatrix}&=\frac{\partial}{\partial x_1}{}^t[A_{13}]\begin{Bmatrix}u_1\\u_2\\u_3\end{Bmatrix}+\frac{\partial}{\partial x_2}{}^t[A_{23}]\begin{Bmatrix}u_1\\u_2\\u_3\end{Bmatrix}+\frac{\partial}{\partial x_3}[A_{33}]\begin{Bmatrix}u_1\\u_2\\u_3\end{Bmatrix}
\end{aligned}$$

where the matrices $[A_{ij}]$ depends on the viscosity factor, the compressibility factor and the frequency. From the 2nd line of eq.(5), one deduces the derivatives of the displacement versus x_2 :

$$\frac{\partial}{\partial x_2}\begin{Bmatrix}u_1\\u_2\\u_3\end{Bmatrix}=[A_{22}]^{-1}\left\{\begin{Bmatrix}\sigma_{21}\\\sigma_{22}\\\sigma_{23}\end{Bmatrix}-\frac{\partial}{\partial x_1}{}^t[A_{12}]\begin{Bmatrix}u_1\\u_2\\u_3\end{Bmatrix}-\frac{\partial}{\partial x_3}[A_{23}]\begin{Bmatrix}u_1\\u_2\\u_3\end{Bmatrix}\right\} \qquad (6)$$

By properly combining these different expressions, it is then possible to obtain the eigenvalue formulation we are looking for by assuming an harmonic dependence of the fields versus x_1 and x_3, replacing the corresponding gradients by $j\omega s_1$ and $j\omega s_3$ with s_i the slowness along x_i :

$$\frac{\partial}{\partial x_2}\begin{Bmatrix}\sigma_{21}\\\sigma_{22}\\\sigma_{23}\\u_1\\u_2\\u_3\end{Bmatrix}=\begin{bmatrix}[\alpha_{11}] & [\alpha_{12}]\\ [A_{22}]^{-1} & [A_{22}]^{-1}(j\omega s_1\,{}^t[A_{12}]+j\omega s_3[A_{23}])\end{bmatrix}\begin{Bmatrix}\sigma_{21}\\\sigma_{22}\\\sigma_{23}\\u_1\\u_2\\u_3\end{Bmatrix} \qquad (7)$$

with

$$[\alpha_{11}]=\left[(j\omega s_1[A_{12}]+j\omega s_3\,{}^t[A_{23}])[A_{22}]^{-1}\right]$$

$$[\alpha_{12}]=\begin{bmatrix}j\omega\rho[I]-j\omega s_1\left([A_{12}][A_{22}]^{-1}\left(j\omega s_1\,{}^t[A_{12}]+j\omega s_3[A_{23}]\right)+j\omega(s_1[A_{11}]+s_3[A_{13}])\right)\\ -j\omega s_3\left({}^t[A_{23}][A_{22}]^{-1}\left(j\omega s_1\,{}^t[A_{12}]+j\omega s_3[A_{23}]\right)+j\omega(s_1[A_{13}]+s_3[A_{33}])\right)\end{bmatrix}$$

The main difficulty introduced by the proposed development consists in the frequency dependence of matrices $[A_{ij}]$, requiring the computation of these matrices for each frequency point. The way these relations can be used to compute the Green's function of the medium are detailed in [4]. This Green's function can be used for computing the actual response of infinitely periodic SAW and Bulk Acoustic Wave (BAW) devices using different approaches [5]. In this work, we derive the well-known effective permittivity from the Green's function which is then used to compute an harmonic admittance, neglecting the mechanical contribution of the electrodes. This approach first used in [6] for interface wave computations enables one to check the influence of viscous fluids on almost any kind of wave as shown in the next section. This approach inspired from Blötekjaër *& al* (see refs. In [6]) provides an harmonic admittance enabling the theoretical characterisation of non-viscous and viscous fluids on the excitation/detection of any coupled SAW on any layered wave guide. The case of water is treated below for academic purposes.

III. Calculation results

Computations have been performed for different kind of wave polarisation on the most used piezoelectric substrates, i.e. quartz, lithium tantalate and lithium niobate. For lithium tantalate and lithium niobate, Rayleigh as well as leaky SAW have been considered. For quartz, Rayleigh and surface transverse waves have been taken into account. For (YXl)/36° quartz, the influence of water load on acoustic plate modes (APM) has been also simulated for both viscous and non viscous water. Only a part of the obtained results are reported below. Figure 1 shows the general geometry of the problem.

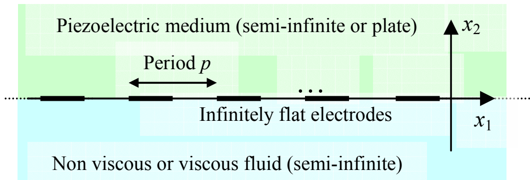
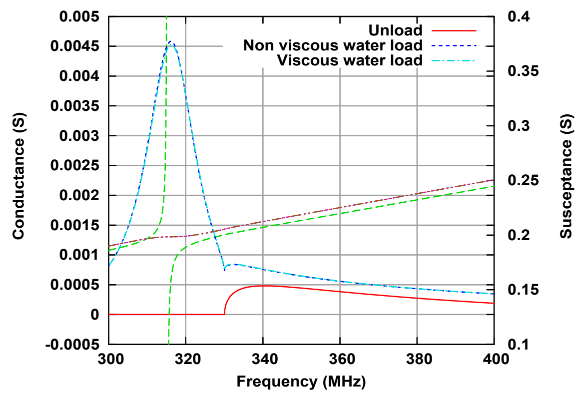
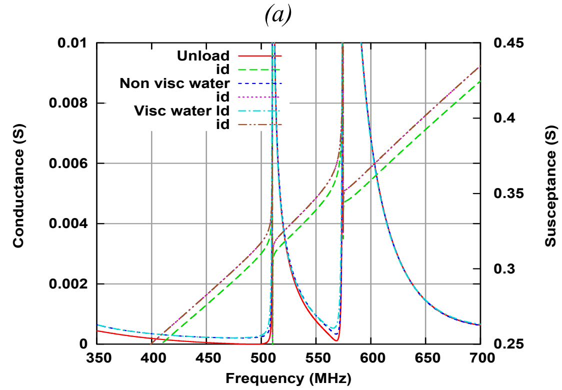


Fig.1 Scheme of the typical geometry considered for simulations

For the sake of consistence, a metal ratio equal to 0.5 was considered for all computations, and the period fixed to 5µm (acoustic wavelength close to 10µm). The viscosity of water was taken equal to 1 centiPoise [6]. Figure 2 shows the results obtained for quartz, expended in 2 sections to magnify the different contributions to the harmonic admittance.



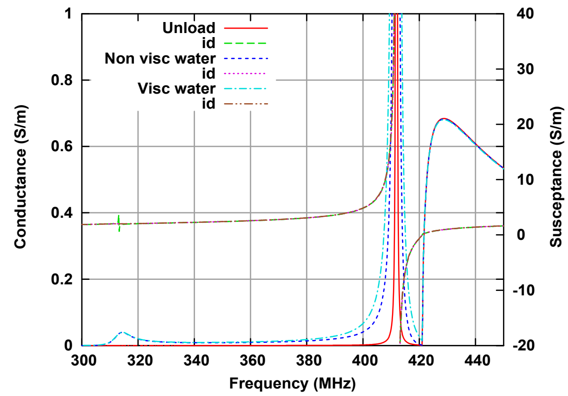
(a)



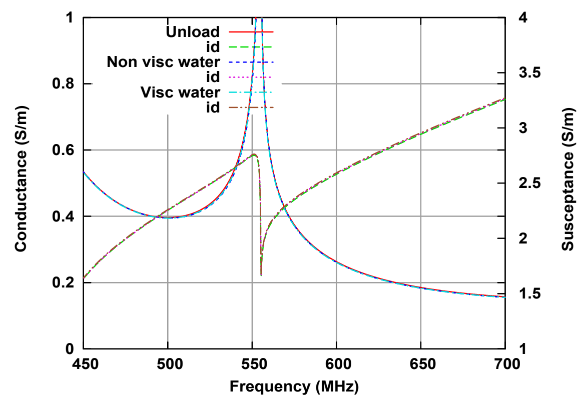
(b)

Fig.2 Harmonic admittance of AT-cut quartz (a) Rayleigh wave and SSBW (b) shear and longitudinal radiated bulk waves

On this graph, the damping of Rayleigh wave by water load is clearly seen. Also a small influence is shown on bulk waves radiate from the surface. Almost no difference is found between viscous and non viscous water loaded admittance. Figure 3 shows the same computations performed for lithium tantalate (YX*l*)/36°.



(a)

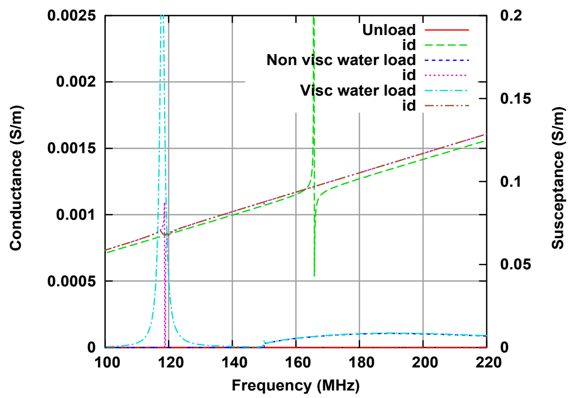


(b)

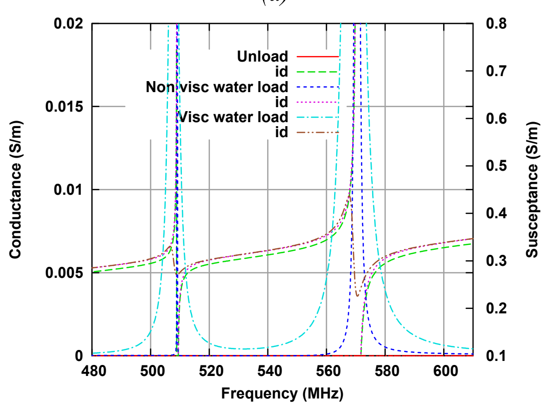
Fig.3 Harmonic admittance of lithium tantalate (a) Rayleigh wave, Leaky SAW and SSBW (b) longitudinal radiated bulk waves

In that case also, that dramatic influence of water load on the Rayleigh wave is emphasized. It is also shown that the water damping also affects the leaky SAW, yielding a significant increase of the conductance peak (and then decreasing the Q factor of the pseudo-mode). One can explain this result by the fact that the leaky SAW on lithium tantalite is not a pure shear wave, exhibiting apart non zero displacement components normal and parallel to the surface. Also the influence of the viscosity is clearly emphasized on this graph, increasing the damping effect on the leaky SAW contribution. Of course, such an influence is not observed so obviously on SSBW contribution since a significant part of their energy is emitted toward the bulk. Similar results are found in the case of lithium niobate cuts exhibiting the same kind of spectrum distribution (Rayleigh, leaky SAW and longitudinal radiated bulk waves).

Finally, the case of APM was also investigated, still considering quartz (YX*l*)/36° and a plate thickness of 2µm, enabling a good mode separation to easily analyse each contribution. The corresponding curves are plotted in fig.4. In fig.4(*a*), one can remark the frequency shift of the first Lamb mode due to water load under the water frequency cut-off. The mode does not then exhibit any loss. Taking into account viscosity generates losses on this mode even if it is well identify as a non leaky propagation. Above the cut-off frequency, a permanent non zero conductance informs one that the device always radiates acoustic energy in the fluid.



(a)



(b)

Fig.4 Harmonic admittance of AT-cut quartz
(a) First Lamb mode and water cut-off
(b) shear and elliptic polarization APM

Figure 4(*b*) shows a close view of the two next plate mode contributions under the same conditions. One can see that the influence of water strongly depend on the mode and its acoustic polarisation. For instance, the two regarded modes in fig. 4(*b*) are assumed to mainly exhibit a shear polarisation. However, the water damping yields more losses for the second than for the first. In both cases, taking viscosity into account causes much larger losses as in the case of the leaky SAW on lithium tantalate. These results show that even for poorly viscous fluid like water, one should not neglect the leakage effects generated by viscosity, more particularly for waves classically considered as shear polarized propagation weakly affected by fluid loads.

IV. Conclusion

A model describing the propagation of acoustic waves in fluids exhibiting or not viscosity properties. The use of an harmonic admittance enables one to determine the influence of non viscous and viscous water load on various combination of waves/substrates. The damping of Rayleigh wave due to the radiation of the displacement field component normal to the guiding surface is clearly pointed out. The effect of the viscosity on this wave appears rather small, since the water damping is predominant in that case. However, the influence of viscosity appears for leaky SAW and also for plate modes exhibiting quasi shear polarisation, yielding an additional leakage phenomenon generally neglected in most of the previous analysis of water damping on SAW. Even Lamb waves assumed propagating under the frequency cut-off exhibit losses due to viscosity which should not be neglected for practical uses. The next development will consists in considering the effect of passivation layers for devices loaded by conductive fluids, and the simulation of more complex devices using mixed finite element analysis and boundary element methods [5].

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